APPLIED RESEARCH LABORATORIES
INTEROFFICE MEMORANDUM

DATE: 15 August 2019

FROM : Brent Renfro
TO : Files
SUBJECT : Clarifying Continuous GPS Time

At the public documents Interface Control Working Group (ICWG) meeting in September 2018, Karl Kovach (Aerospace), Miquela Stein, and I accepted an action to propose a definition for a continuous GPS time (GPST) and how such a definition could be used within the public signal interface specifications. Anthony Flores of Engility has been tasked with shepherding this proposal through the document revision process.

This memo is meant to establish the groundwork for the effort. The following topics are covered.

- A definition of GPST and corresponding proposed wording for IS-GPS-200.
- Proposed text for describing how navigation message quantities are converted to GPST.
- Selected examples of how existing IS-GPS-200 text could be changed to take advantage of the definition of GPST.
- A list of all the places in IS-GPS-200 that would be modified.

Once there is agreement on these points, it will be prudent to engage a wider group of subject matter experts (SMEs) prior to undertaking the effort to re-work multiple locations in IS-GPS-200, IS-GPS-705, and IS-GPS-800.

Definition of GPST

The three of us working on this action item have come to agreement that GPS time – as currently defined in IS-GPS-200 –is already the continuous GPS time that we seek. There are no discontinuities in GPS time.

We have also come to further agreement that what we are really seeking is a simple representation of GPS time that has only one unit (i.e., seconds). This simple representation is distinctly different from the legacy representation of GPS time which is already embodied in IS-GPS-200. The existing representation is more complex by way of employing two units (i.e., weeks and seconds of week).

Explaining Two Representations of GPS Time (GPST)

We recommend IS-GPS-200 should explain as early as possible that it uses two representations of GPS time (GPST). The two representations, and the rationale for using each of them, follow:

a. **GPST(WN+SOW)** GPST represented in terms of the elapsed number of whole weeks (WN) since the start of GPST and the elapsed integer and fractional seconds of the current week (SOW), where both the WN and SOW units are required to unambiguously specify a particular point in time. This representation is abbreviated as GPST(WN+SOW). This is the most common GPST representation in the LNAV message.

b. **<u>GPST(sec)</u>** GPST represented in terms of the elapsed integer and fractional seconds since the start of GPST. Only the unit of seconds is required to unambiguously specify a particular point in time. This representation is abbreviated as GPST(sec). This is simplest GPST representation for algebraically manipulating the parameters in the LNAV message.

Specific Changes to Paragraph 3.3.4 of IS-GPS-200 ("GPS Time and SV Z-Count")

Propose revising first paragraph of IS-GPS-200 3.3.4 as follows.

- - - Begin Changes to Section 3.3.4. Note: New text is in blue; removed text is in red.

3.3.4 GPS Time (GPST) and SV Z-Count.

GPS time (GPST) is established by the Control Segment and is referenced to Coordinated Universal Time (UTC) as maintained by the U.S. Naval Observatory (UTC (USNO)) zero time-point defined as midnight on the night of January 5, 1980/morning of January 6, 1980. The largest units used in stating GPST ~~time~~ are epochs, where one 10-bit LNAV epoch is equal to 1024 weeks and one 13-bit CNAV epoch is equal to 8192 weeks. The next largest unit is one week defined as 604,800 seconds. GPST ~~time~~ may differ from UTC because GPST ~~time~~ shall be a continuous time scale, while UTC is corrected periodically with an integer number of leap seconds. There also is an inherent but bounded drift rate between the UTC and GPST ~~time~~ scales. The OCS shall control the GPST ~~time~~ scale to be within one microsecond of UTC (modulo one second).

3.3.4.1 GPST Representations.

This IS uses two representations of GPST. The two representations, and the rationale for using each of them, follow:

a. GPST is represented in terms of the elapsed number of whole weeks (WN) since the start of GPST <u>and</u> the elapsed integer and fractional seconds of the current week (SOW), where both the WN number with units of weeks and the SOW number with units of seconds are required to unambiguously specify a particular point in time. This representation is abbreviated as GPST(WN+SOW). The broadcast NAV data does not, however, provide the full WN for any point in time; it only provides a truncated version of the WN due to bit allocation limits. This is the most common form of GPST representation in the broadcast NAV data.

b. GPST is represented in terms of the elapsed integer and fractional seconds since the start of GPST. With this representation, only one number with units of seconds is required to unambiguously specify a particular point in time. This representation is abbreviated as GPST(sec). This is the simplest GPST representation to use for algebraically manipulating the parameters in the broadcast NAV data.

3.3.4.1 Converting Between GPST Representations.

Converting GPST(WN+SOW) to GPST(sec) requires knowledge of the WN. Due to the limited number of bits available in the broadcast NAV data messages, the broadcast NAV data only provides a truncated representation of the WN directly to users. The LNAV data uses a maximum of 10 bits to represent the least significant portion of the WN (effective range 0-1023) for each point in time while the CNAV data uses a maximum of 13 bits to represent the least significant portion of the WN (effective range 0-8191) for each point in time. For any GPST(WN+SOW) point in time identified by the broadcast NAV data, GPS users must therefore employ their own knowledge as to which 1024-week LNAV GPST epoch applies to that GPST(WN+SOW) time point in the LNAV data message or their own knowledge as to which 8192-week CNAV GPST epoch is associated with that GPST(WN+SOW) time point in the CNAV data.

Table 3-VIIIa provides the date ranges for the first three 10-Bit (1024 Week) LNAV GPST epochs in the LNAV data message, and Table 3-VIIIb provides the date ranges for the first two 13-Bit (8192 Week) CNAV GPST epochs in the CNAV data message.

Table 3-VIIIa. Dates for First Four 10-Bit (1024 Week) LNAV GPST Epochs

10-Bit GPST Epoch	Starting Week	Begin Date	End Date
0	0	6 January 1980	21 August 1999
1	1024	22 August 1999*	6 April 2019
2	2048	7 April 2019*	20 November 2038
3	3072	21 November 2038*	6 July 2058

* Actual UTC calendar date will generally be earlier due to accumulated leap seconds

Table 3-VIIIb. Dates for First Two 13-Bit (8192 Week) CNAV GPST Epochs

13-Bit GPST Epoch	Starting Week	Begin Date	End Date
0	0	6 January 1980	5 January 2137
1	8192	6 January 2137*	6 January 2294

* Actual UTC calendar date will generally be earlier due to accumulated leap seconds

Furthermore, and also due to the limited number of bits available in the broadcast NAV data messages, the broadcast NAV data occasionally uses only 8 bits to provide an even more truncated representation of the least significant portion of the WN (effective range 0-255) for certain specific points in time. The 10-bit, 13-bit, and 8-bit reference points in time are listed in Table 3-IXa for the LNAV data message and in Table 3-IXb for the CNAV data message. These tables provide the names of the GPST(WN+SOW) quantities, the truncated WN bit sizes, locations in the NAV messages where the quantities appear, and the corresponding GPST(sec) quantity.

Table 3-IXa. LNAV Reference Time Quantities and Corresponding GPST(sec) Quantities

Reference Time	GPST(WN+SOW)		#bits of WN	Location	Corresponding GPST(sec)
	WN_x	SOW			
Subframe Transmit End	WN_{trans}	TOW	10	subframe 1	$GPST_{trans}$
Clock	WN_{trans}	toc	10	subframe 1	$GPST_{oc}$
Ephemeris	WN_{trans}	toe	10	subframe 1. 2	$GPST_{oe}$
Almanac	WN_{oa}	toa	8	subframe 4, 5	$GPST_{oa}$
UTC	WN_t	tot	8	subframe 4, page 18	$GPST_{ot}$
UTC leap second	WNLSF	DN	8	subframe 4, page 18	$GPST_{LSF}$

WN_{trans} is not the full WN_{trans}, it is the 10 least significant bits of the full WN_{trans} and is only provided in subframe 1

Table 3-IXb. CNAV Reference Time Quantities and Corresponding GPST(sec) Quantities

Reference Time	GPST(WN+SOW)		#bits of WN	Location	Corresponding GPST(sec)
	WN_x	SOW			
Message Transmit End	WN_{trans}	TOW	13	MT 10	$GPST_{trans}$
Clock	WN_{trans}	toc	13	MT 10, 30-37	$GPST_{oc}$
Ephemeris	WN_{trans}	toe	13	MT 10, 11	$GPST_{oe}$
Almanac	WN_{oa}	toa	13	MT 12, 31, 37	$GPST_{oa}$
UTC	WN_{ot}	t_{ot}	13	MT 33	$GPST_{ot}$
UTC leap second	WNLSF	DN*	13	MT 33	$GPST_{LSF}$

Propagation	WN_{op}	t_{op}	8	MT 30	$GPST_{op}$
Earth Orientation	–	teop	–	MT 30	$GPST_{EOP}$
Differential Correction	–	$t_{op\text{-}D}$	–	MT 13,14	$GPST_{op\text{-}D}$
Differential Correction	–	t_{OD}	–	MT 13,14	$GPST_{OD}$
GGTO	WN_{GGTO}	t_{GGTO}	13	MT 35	$GPST_{GGTO}$

WN_{trans} is not the full WN_{trans}, it is the 13 least significant bits of the full WN_{trans} and is only provided in MT 10

Fundamentally, and most directly, the GPST(sec) equivalent of a given GPST(WN+SOW) quantity is defined by:

GPST(sec) = Full WN * 604800 + SOW

When dealing with truncated WN_x terms in the broadcast LNAV/CNAV parameters, the GPST(sec) equivalent of a given quantity represented in GPST(WN+SOW) terms may be derived by:

$$GPST_x = (WN_x + WN_{epoch,x}) * 604800 + SOW_x.$$

A slightly different form of the above equation applies to the computation of the current subframe transmit end time and/or the current message transmit end time ($GPST_x = GPST_{trans}$) due to the use of a broadcast time of week (TOW) -count that updates with every subframe/message in lieu of a fixed broadcast SOW value. Those equations for $GPST_{trans}$ in units of seconds are:

<u>LNAV</u>

$$GPST_{trans} = (WN_{trans} + WN_{epoch,trans}) * 604800 + TOW * 6$$

<u>CNAV</u>

$$GPST_{trans} = (WN_{trans} + WN_{epoch,trans}) * 604800 + TOW * 12 \quad \text{(for L2C)}$$
$$GPST_{trans} = (WN_{trans} + WN_{epoch,trans}) * 604800 + TOW * 6 \quad \text{(for L5)}$$

The term $WN_{epoch,x}$ (e.g., $WN_{epoch,trans}$) in the above equations arises from the fact that the LNAV/CNAV messages do not provide the full WN for GPST. The term $WN_{epoch,x}$ is calculated based the number of bits in the associated broadcast truncated WN_x value. The user must provide the knowledge of the number of GPST epochs (10-bit epochs, 8-bit epochs, or 13-bit epochs as appropriate) that have passed since the beginning of GPST. The appropriate $WN_{epoch,x}$ value may be calculated as follows.

<u>10-bit Epochs for 10-bit WN_x in LNAV:</u>

See Table 3-VIIIa and Table 3-IXa. Mathematically,

GPST epoch = (full GPS week – mod(full GPS week,1024)) / 1024

$WN_{epoch,x}$ = GPST epoch * 1024

<u>8-bit Epochs for 8-bit WN_x in LNAV/CNAV:</u>

See Table 3-IXa and 3-IXb.

$WN_{epoch,x}$ = GPST epoch * 1024 + (WN_{trans} – mod(WN_{trans},256)) + rollover_adjust

where WN_{trans} is the week number from the subframe 1 transmitted in the same frame as WN_x and rollover_adjust is derived by the following process.

rollover_adjust = 0.
rollover_check = WN_x – mod(WN_{trans},256).

if (rollover_check<-128) rollover_adjust = 256.
if (rollover_check>128) rollover_adjust = -256,

<u>13-bit Epochs for 13-bit WN_x in CNAV:</u>

See Table 3-VIIIb and Table 3-IXb.

GPST epoch = (full GPS week – mod(full GPS week,8192)) / 8192

$WN_{epoch,x}$ = GPST epoch * 8192

3.3.4.3 Relating GPST to UTC.

The NAV data contains the requisite data for relating GPST~~ime~~ to UTC. The accuracy of this data during the transmission interval shall be such that it relates GPST~~ time~~ (maintained by the MCS of the CS) to UTC (USNO) within 20 nanoseconds (one sigma). This data is generated by the CS; therefore, the accuracy of this relationship may degrade if for some reason the CS is unable to upload data to a SV. At this point, it is assumed that alternate sources of UTC are no longer available, and the relative accuracy of the GPST/UTC relationship will be sufficient for users. Range error components (e.g. SV clock and position) contribute to the GPST time transfer error, and under normal operating circumstances (two frequency time transfers from SV(s) whose navigation message indicates a URA of eight meters or less), this corresponds to a 28 nanosecond (one sigma) apparent uncertainty at the SV. Propagation delay errors and receiver equipment biases unique to the user add to this time transfer uncertainty.

3.3.4.4 Z-Count Time.

In each SV the X1 epochs of the P-code offer a convenient unit for precisely counting and communicating time. Time stated in this manner is referred to as Z-count, which is given as a binary number consisting of two parts as follows:

a. The binary number represented by the 19 least significant bits of the Z-count is referred to as the TOW count and is defined as being equal to the number of X1 epochs that have occurred since the transition from the previous week. The count is short-cycled such that the range of the TOW-count is from 0 to 403,199 X1 epochs (equaling one week) and is reset to zero at the end of each week. The TOW-count's zero state is defined as that X1 epoch which is coincident with the start of the present week. This epoch occurs at (approximately) midnight Saturday night-Sunday morning, where midnight is defined as 0000 hours on the UTC scale which is nominally referenced to the Greenwich Meridian. Over the years the occurrence of the "zero state epoch" may differ by a few seconds from 0000 hours on the UTC scale since UTC is periodically corrected with leap seconds while the TOW-count is continuous without such correction. To aid rapid ground lock-on to the P-code signal, a truncated version of the TOW-count, consisting of its 17 most significant bits, is contained in the hand-over word (HOW) of the L1 and L2 NAV data (D(t)) stream; the relationship between the actual TOW-count and its truncated HOW version is illustrated by Figure 3-16.

b. The most significant bits of the Z-count are a binary representation of the sequential number assigned to the current GPS week (see paragraph 6.2.4).

The NAV data broadcast by each satellite contains the requisite data for relating the satellite's Z-count time to GPST.

- - - End Changes to Section 3.3.4.

Examples of Proposed Changes to IS-GPS-200 that Utilize GPST

Figure 1 illustrates how GPST could be utilized in IS-GPS-200 20.3.3.3.3.1. Figure 2 illustrates how GPST could be utilized in IS-GPS-200 20.3.3.5.2.4.

Change
"t" to "$GPST_t$",
"t_{sv}" to "$GPST_{sv}$", and
"t_{oc}" to "$GPST_{oc}$"
where circled

20.3.3.3.3.1 User Algorithm for SV Clock Correction.

The polynomial defined in the following allows the user to determine the effective SV PRN code phase offset referenced to the phase center of the antennas (Δt_{sv}) with respect to GPS system time (t) at the time of data transmission. The coefficients transmitted in subframe 1 describe the offset apparent to the two-frequency user for the interval of time in which the parameters are transmitted. This estimated correction accounts for the deterministic SV clock error characteristics of bias, drift and aging, as well as for the SV implementation characteristics of group delay bias and mean differential group delay. Since these coefficients do not include corrections for relativistic effects, the user's equipment must determine the requisite relativistic correction. Accordingly, the offset given below includes a term to perform this function.

The user shall correct the time received from the SV with the equation (in seconds)

$$t = t_{sv} - \Delta t_{sv} \qquad (1)$$

where

t = GPS system time (seconds),

t_{sv} = effective SV PRN code phase time at message transmission time (seconds),

Δt_{sv} = SV PRN code phase time offset (seconds).

The SV PRN code phase offset is given by

$$\Delta t_{sv} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_r \qquad (2)$$

where

a_{f0}, a_{f1} and a_{f2} are the polynomial coefficients given in subframe 1, t_{oc} is the clock data reference time in seconds (reference paragraph 20.3.4.5), and Δt_r is the relativistic correction term (seconds) which is given by

Change "20.3.4.5" to "20.3.4.5.1"

$$\Delta t_r = F e \sqrt{A} \sin E_k.$$

The orbit parameters (e, $\sqrt{A}$, E_k) used here are described in discussions of data contained in subframes 2 and 3, while F is a constant whose value is

$$F = \frac{-2\sqrt{\mu}}{c^2} = -4.442807633 (10)^{-10} \frac{\text{sec}}{\sqrt{\text{meter}}},$$

where

Figure 1a. Application of GPST to IS-GPS-200 Section 20.3.3.3.3.1 (1 of 2)

$\mu = 3.986005 \times 10^{14} \frac{\text{meters}^3}{\text{second}^2}$ = value of Earth's universal gravitational parameters

$c = 2.99792458 \times 10^8 \frac{\text{meters}}{\text{second}}$ = speed of light.

Note that equations (1) and (2), as written, are coupled. While the coefficients a_{f0}, a_{f1} and a_{f2} are generated by using GPS time as indicated in equation (2), sensitivity of t_{sv} to t is negligible. This negligible sensitivity will allow the user to approximate t by tsv in equation (2). The value of t must account for beginning or end of week crossovers. That is, if the quantity t - t_{oc} is greater than 302,400 seconds, subtract 604,800 seconds from t. If the quantity t - t_{oc} is less than -302,400 seconds, add 604,800 seconds to t.

REMOVE

The control segment will utilize the following alternative but equivalent expression for the relativistic effect when estimating the NAV parameters:

$$\Delta t_r = -\frac{2\,\vec{R}\bullet\vec{V}}{c^2}$$

where

$\vec{R}$ is the instantaneous position vector of the SV,

$\vec{V}$ is the instantaneous velocity vector of the SV, and

c is the speed of light. (Reference paragraph 20.3.4.3).

It is immaterial whether the vectors $\vec{R}$ and $\vec{V}$ are expressed in earth-fixed, rotating coordinates or in earth-centered, inertial coordinates.

Figure 1b. Application of GPST to IS-GPS-200 Section 20.3.3.3.3.1 (2 of 2)

Change
"t_E" to "$GPST_E$",
"t_{UTC}" to "$GPST_{UTC}$", and
"t_{OT}" to "$GPST_{OT}$"
where circled

starts at six hours prior to the effectivity time and ends at six hours after the effectivity time, the UTC/GPS-time relationship is given by

$$t_{UTC} = (t_E - \Delta t_{UTC}) \text{ [modulo 86400 seconds]}$$

Replace with "($GPST_E$ – $GPST_{ot}$)"

where t_{UTC} is in seconds and

Δt_{UTC} = $\Delta t_{LS} + A_0 + A_1$ (~~t_E - t_{ot} + 604800 (WN - WN_t)~~), seconds;

t_E = GPS time as estimated by the user after correcting t_{SV} for factors described in

paragraph 20.3.3.3.3 as well as for selective availability (SA) (dither) effects;

Δt_{LS} = delta time due to leap seconds;

A_0 and A_1 = constant and first order terms of polynomial;

Replace with "20.3.4.5.1"

t_{ot} = reference time for UTC data (reference ~~20.3.4.5~~);

~~WN = current week number (derived from subframe 1);~~

~~WN_t = UTC reference week number.~~

Replace with "$GPST_{SEC}$"

The estimated GPS time (t_E) shall be in ~~seconds relative to end/start of week~~. During the normal and short-term extended operations, the reference time for UTC data, t_{ot}, is some multiple of 2^{12} seconds occurring approximately 70 hours after the first valid transmission time for this UTC data set (reference 20.3.4.5). ~~The reference time for UTC data (t_{ot}) shall be referenced to the start of that week whose number (WN_t) is given in word eight of page 18 in subframe 4. The WN_t value consists of eight bits which shall be a modulo 256 binary representation of the GPS week number (see paragraph 6.2.4) to which the t_{ot} is referenced. The user must account for the truncated nature of this parameter as well as truncation of WN, WN_t, and WN_{LSF} due to rollover of full week number (see paragraph 3.3.4(b)).~~ REMOVE The CS shall manage these parameters such that the absolute value of the difference between the untruncated WN and WN_t values shall not exceed 127.

b. Whenever the user's current time falls within the time span of six hours prior to the effectivity time to six hours after the effectivity time, proper accommodation of the leap second event with a possible week number transition is provided by the following expression for UTC:

$$t_{UTC} = W\text{[modulo } (86400 + \Delta t_{LSF} - \Delta t_{LS})\text{], seconds;}$$

where

$$W = (t_E - \Delta t_{UTC} - 43200) \text{ [modulo 86400]} + 43200 \text{, seconds;}$$

and the definition of Δt_{UTC} (as given in 20.3.3.5.2.4a above) applies throughout the transition period. Note that when a leap second is added, unconventional time values of the form 23:59:60.xxx are encountered. Some user equipment may be designed to approximate UTC by decrementing the running count of time within several seconds after the event, thereby promptly returning to a proper time indication. Whenever a leap second event is encountered, the user equipment must consistently implement carries or borrows into any year/week/day counts.

Figure 2. Application of GPST to IS-GPS-200 Section 20.3.3.5.2.4

List of IS-GPS-200 Sections to Consider for Modification

The following sections of IS-GPS-200 will need modification as part of the introduction of GPST.

Section Number	**Items that May Need Modification**
3.3.4 GPS Time and SV Z-Count	Introduce new definition
6.1 Acronyms table	Add definition of GPST
20.3.3.3.3.1 User Algorithm for SV Clock Correction	SV clock correction eqs 1 and 2, t_{oc}
20.3.3.4.3 User Algorithm for SV Ephemeris Determination	t_{oe}, Table 20-IV: t_{oe}, t_k eq
20.3.3.4.3.4 Geometric Range	t_R and t_T
20.3.3.4.4 NMCT Validity Time	t_{nmct}, t_{oe}, nmct eq
20.3.3.5.1.5 Almanac Reference Week	WN_a and t_{oa}
20.3.3.5.2.2 Almanac Reference Time	t_{oa}, t_{oe}
20.3.3.5.2.3 Almanac Time Parameters	t, t_{SV}, t_k
20.3.3.5.2.4 Coordinated Universal Time (UTC)	t_{UTC}, t_E, t_{ot}
30.3.3.1.3 User Algorithm for Determination of SV Position	t_{oe}, Table 30-II: t_{oe}, t_k eq
30.3.3.2.1.1 SV Clock Correction	Add pointer to new 20.3.4.5.1-equivalent in 30 to relate t_{oc} to $GPST_{oc}$
30.3.3.2.1.2 Data Predict Time of Week	Add pointer to new 20.3.4.5.1-equivalent in 30 to relate t_{op}/WN_{op} to $GPST_{op}$
30.3.3.2.4 NED Accuracy Estimates	IAURA calculations
30.3.3.3.1.3 Data Predict Week Number	Add pointer to new 20.3.4.5.1-equivalent in 30 to relate t_{op}/WN_{op} to $GPST_{op}$
30.3.3.4.1 Almanac Reference Week	WN_{a-n} and t_{oa}
30.3.3.5.1.1 User Algorithm for Application of the EOP	t_{EOP}, Table 30-VIII: EOP param. eqns.
30.3.3.6.2 UTC and GPS Time	t_{UTC}, t_E, t_{ot}
30.3.3.7.2.1 DC Data Predict Time of Week	t_{op-D}
30.3.3.7.3 Application of Clock-Related DC Data	t, t_{oc}, Δt_{SV} eqn,
30.3.3.7.4 Application of Orbit-Related DC Data	t_{op-D}, t_{oe}, WN_{oe}, t_{OD}, ΔM_0 eqn.
30.3.3.8.2 GPS and GNSS Time	t_E, t_{GNSS}, t_{GGTO}, WN, WN_{GGTO}