Controlling Clocks

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Masterclock, Inc.

A ten-minute tour

If this interests you …

Matsakis, Metrologia 56 (1), March 8, 2019

Free download:

https://iopscience.iop.org/article/10.1088/1681-7575/ab0614

"TMI version"

Matsakis, ION-PTTI 2019

Free download

https://tycho.usno.navy.mil/papers/ts-2019/ControlledClocks.IONPTTI.Matsakis2019.pdf

Goal

Control a clock to be

1. On Time
2. On Frequency

With respect to

- Master Clock, such as UTC(k)
- or GPS
- or other GNSS

The Means to the Goal: Steering

Steer = frequency adjustment

The Gain is the Game

- Gain Vector = $G = (g_x \quad g_y)$
- Steer = -$(g_x \quad g_y)\begin{pmatrix} phase \\ frequency \end{pmatrix}$

(phase = time for purpose of this talk)

Nothing is Perfect

- A steered clock will never exactly mimic the Master
- You can optimize some aspects of its performance
  - Only at the expense of other aspects
- Some performance elements:
  - Steady-state RMS deviation in time
  - Steady-state RMS deviation in frequency
  - Short-term instability due to excessive steering
  - Response to non-stochastic disturbances

One way: LQG = Linear Quadratic Gaussian

Optimal Gains for your priorities

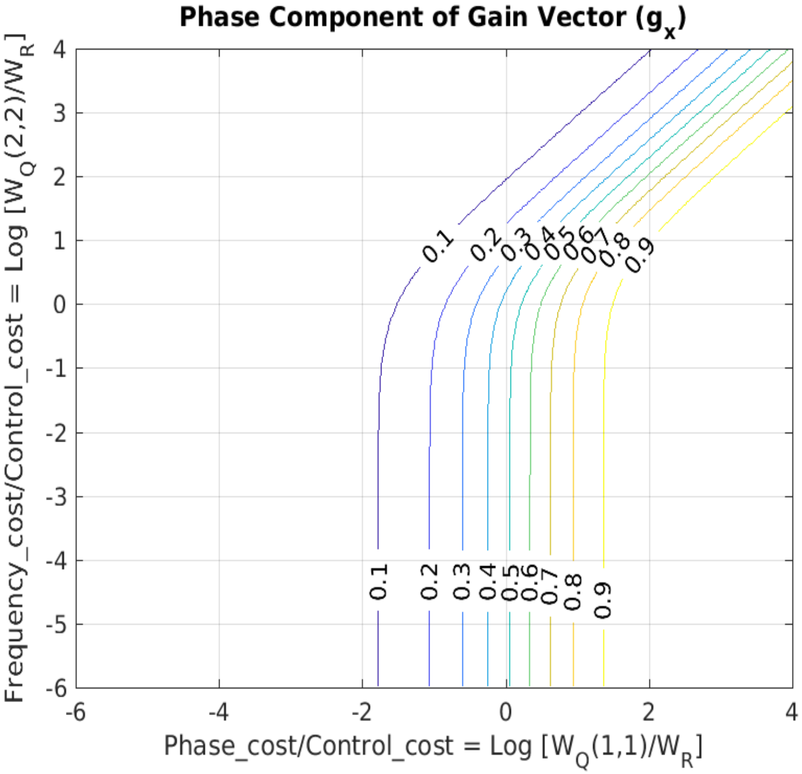
Compromise between

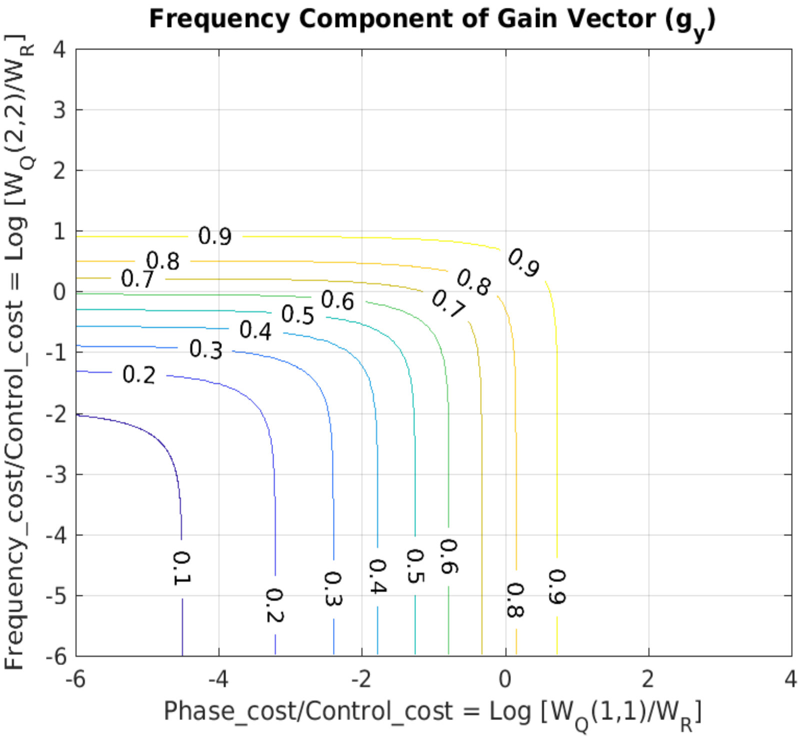
- frequency offset RMS
- phase offset RMS
- steers RMS

Equivalently

- (frequency RMS)/(steers RMS)
- (phase RMS)/(steers RMS)

LQG in two plots





Alternate approach: setting Time Constants

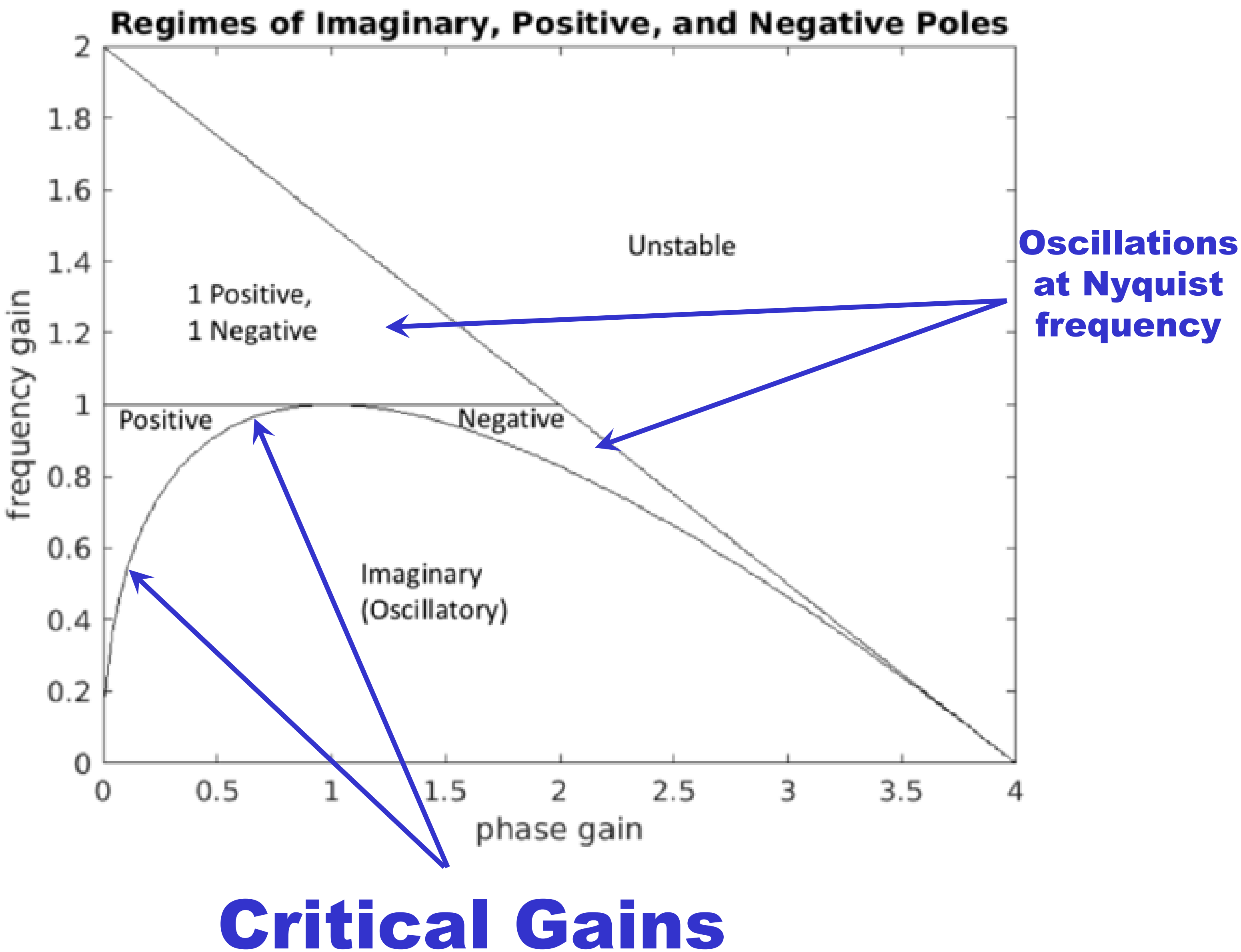
- **How quickly does it recover from:**
  - **Noise fluctuations**
  - **Systematic variations**
  - **Non-Gaussian behavior (jumps, etc.)**
- **Typically two time constants are at work**
  - **Short-term exponential**
  - **Long-term exponential**
- **Can also have [decaying] oscillations**

Critical Gains

Result in only one time constant and no oscillations

- T=Time Constant
- $g_x = (1 - e^{-1/T})^2$
- $g_{\mathrm{y}} = 1 - e^{-2/T}$

Gains and Time Constants



In the “real world”

- Systems need to be super-robust
  - Especially if not monitored or redundant
- Crystal or Rubidium oscillators
  - Example: a controlled clock *at time t*
    - Phase offset =X
    - Frequency offset = 0
    - So steer is $-g_x$*X
    - If controlled clock’s raw frequency shifts by $+g_x$*X
      - The clock oscillator’s frequency drift negates the steering
      - If it happens as random fluctuation, no problem
        - What about a crystal with systematic freq. drift= $+g_x$*X?

PID Controllers

- Use three gains
  - P = Phase gain
  - D =Derivative gain (Frequency gain)
  - I = Integral gain = sum of past Phase deviations
- The Integral gain ("I")
  - can handle systematic freq. drift in underlying clock
    - as in the previous slide
  - can lead to faster convergence
  - BUT the extra gain parameter requires care
    - can lead to instabilities, excessive overshooting

Many ad-hoc PID optimization methods exist

- The Wikipedia lists 6 classes of approaches
- A seventh is analogous to the critical gains:
  - $g_P = 1 - 3e^{-2/T} + 2e^{-3/T}$
  - $g_I = 1 - 3e^{-1/T} + 3e^{-2/T} - e^{-3/T}$
  - $g_D = 1 - e^{-3/T}$
  - T=Time Constant, in units of measurement interval
- My time is up
  - But the work has just begun!

backups

Separation Principle

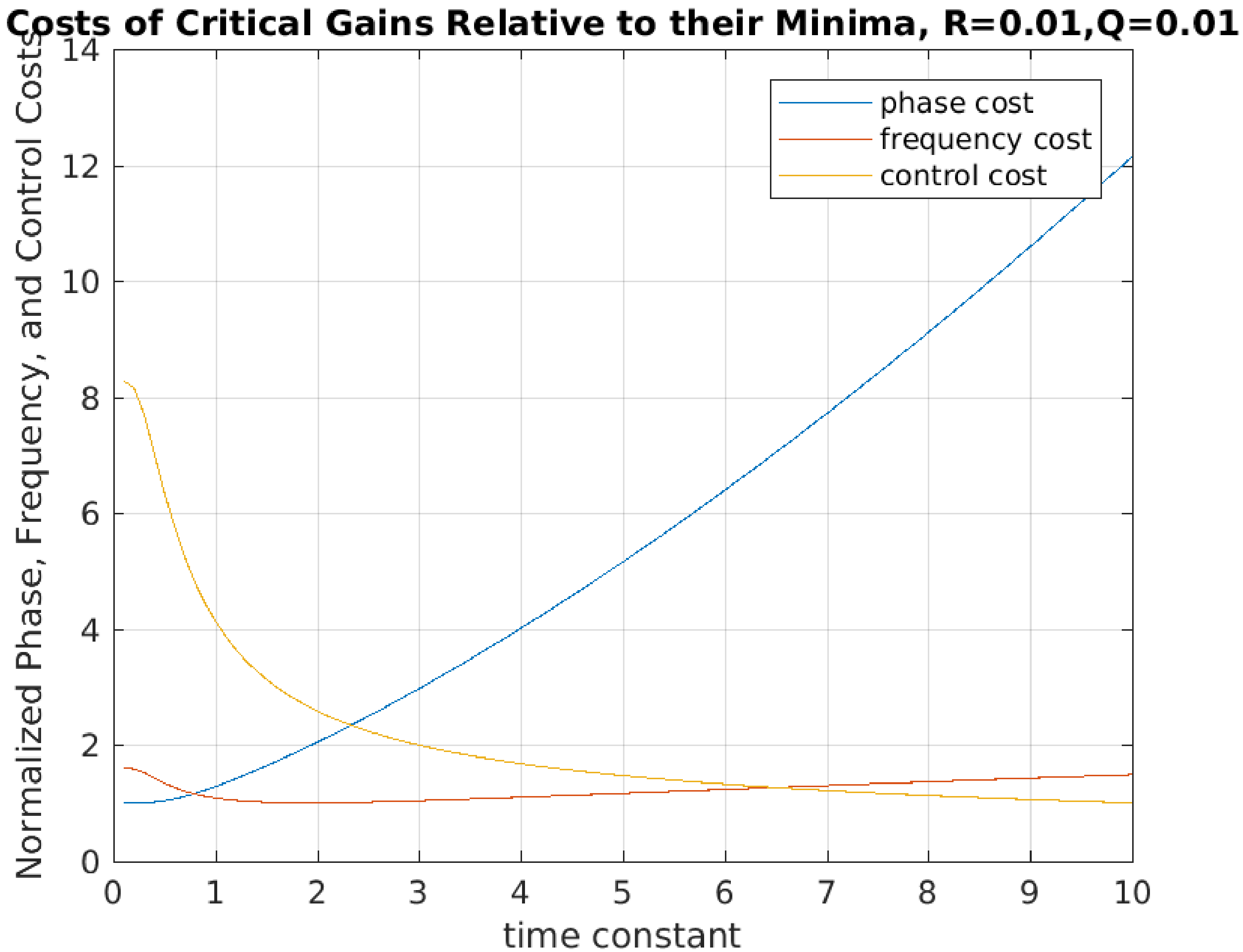
The optimal gain is independent of the state estimate

(i.e. how good your clocks are)

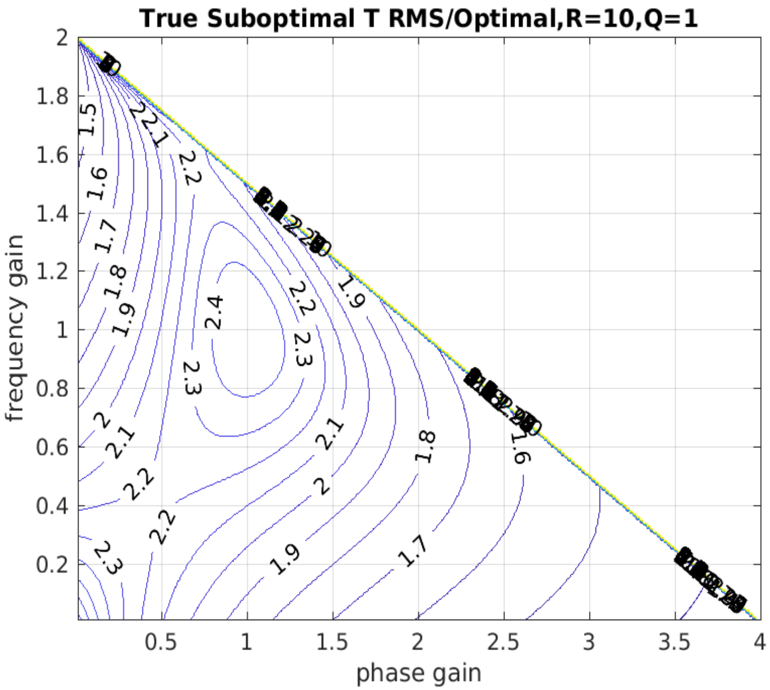
- The calculations for optimality do not use any state estimation parameters

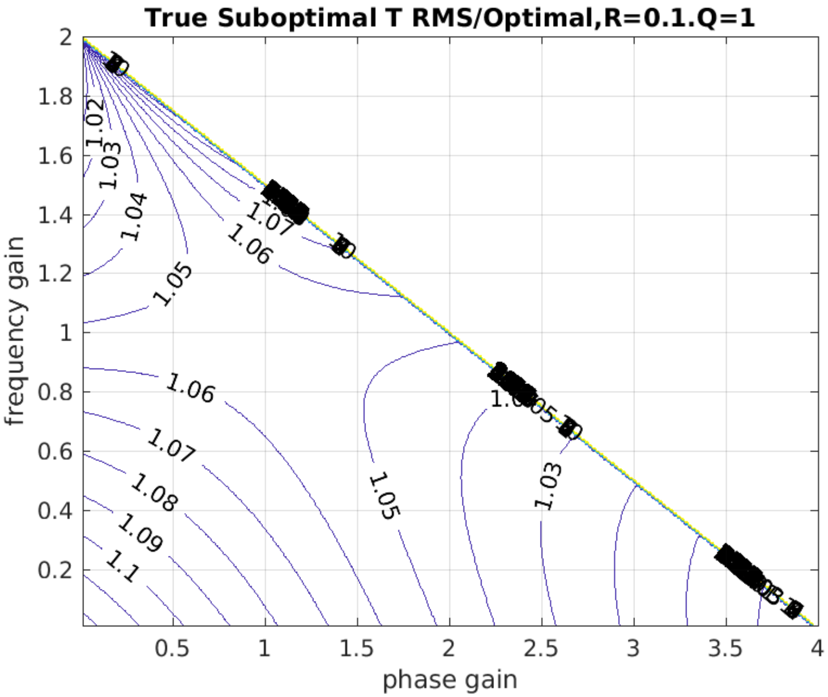
- But you MUST make an optimal state estimate
  - given your data and clock stability

Stabilities associated with critical gains



Suboptimal State Estimation

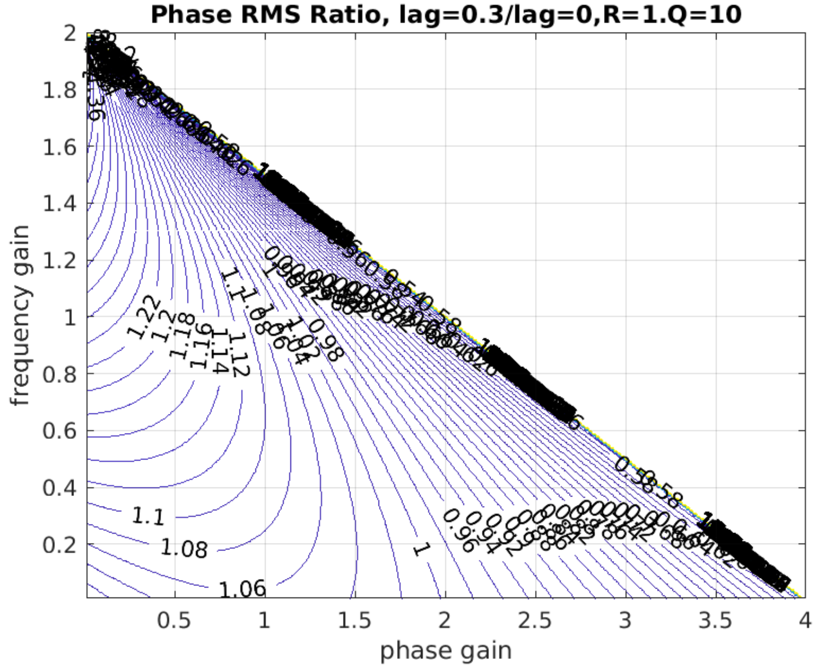


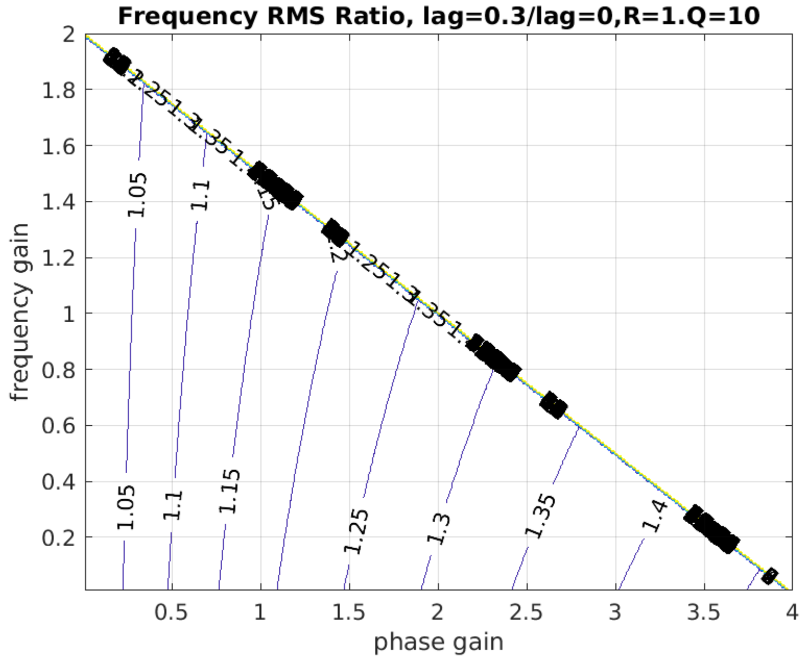


High Meas. Noise (R/Q=10)

Low Meas. Noise (R/Q=0.1)

Effect of 10-day delay in Circular T

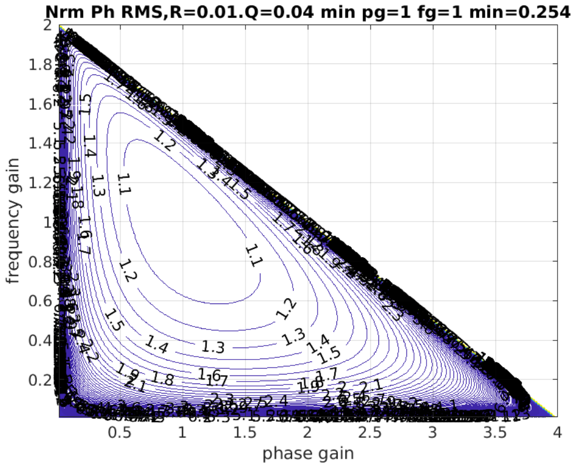


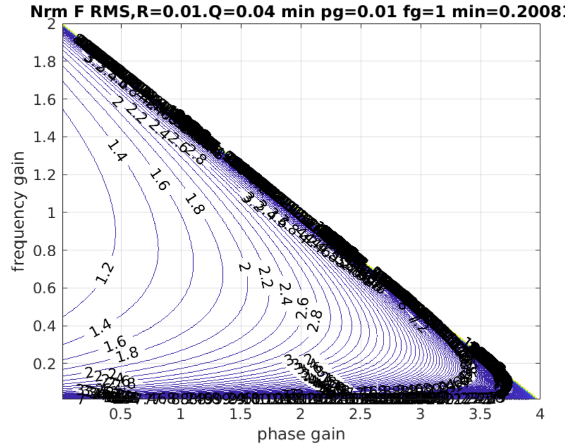


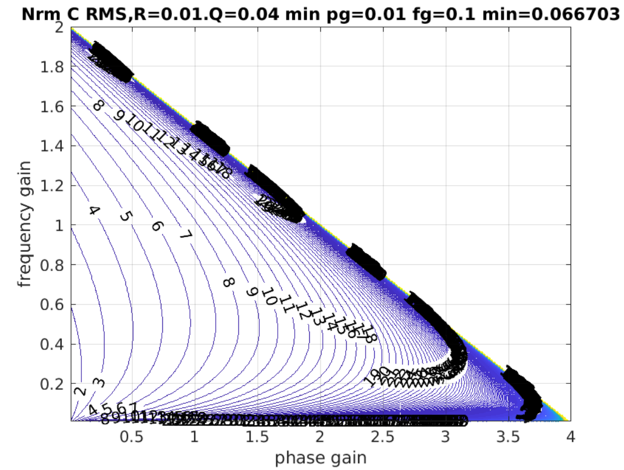
Measurement Noise = 1 ns

Freq Stab = 3 10^{-14} @ 1 month

Minimum RMS as a function of gains







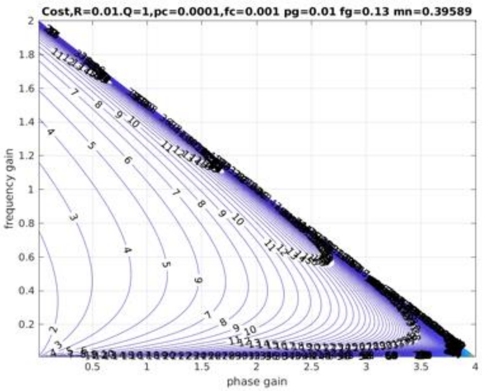
Phase RMS (normalized)

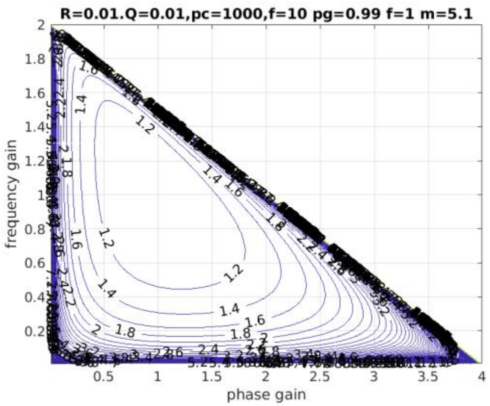
Frequency RMS (normalized)

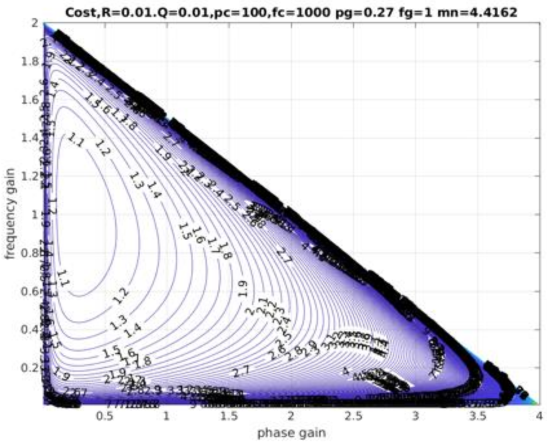
Steer RMS (normalized

- **Contour details depend on assumed noises**
- **LQG finds optimum gains for any linear sum of their variances**
  - **independently of the noises**

Three cost functions







Phase Cost=10^{-4}
Freq Cost=10^{-3}
Steer Cost=1

Phase Cost=1000
Freq Cost=100
Steer Cost=1

Phase Cost=100
Freq Cost=1000
Steer Cost=1

Z-Transforms and Poles

$$p_k = (2 - \tau g_1 - g_2)p_{k-1} - (1 - g_2)p_{k-2} + n_k$$

p_k =state, n_k =noise, τ = spacing

- Desire: exponential behavior for $p_k = p_0 e^{-\tau k/T}$
- Take Z-transform of both sides
- Leads to equations for gains g_1 and g_2
    - *Oscillatory* solutions have imaginary T's
    - Others have *two* real decay times T
    - A very few *critical gains* have one decay time